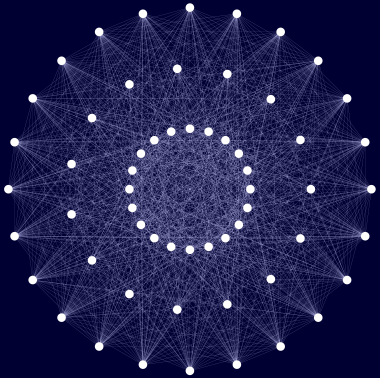
$\frac{3}{2}$-Generation of Finite Groups



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Young Researchers in Mathematics

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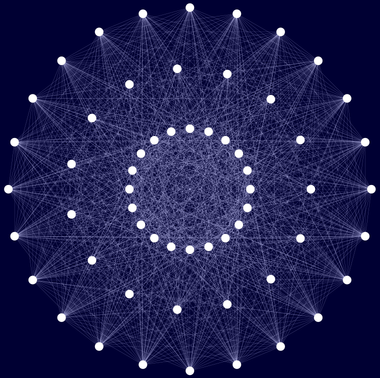

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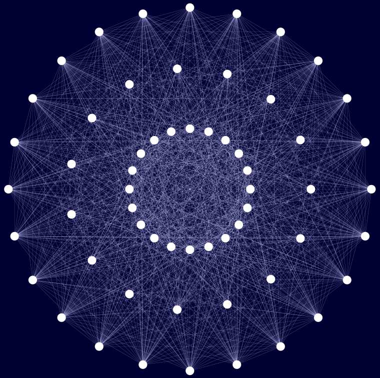

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Alternating groups are 2-generated:
– if n is odd $A_n = \langle (1\,2\,3), (1\,2\,\ldots\,n) \rangle$
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Are other important families of finite groups 2-generated?

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Theorem (Steinberg, 1962; Aschbacher & Guralnick, 1984)

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Netto's Conjecture

If we arbitrarily select two or more substitutions of n elements, it is to be regarded as extremely probable that the group of lowest order which contains these is the symmetric group, or at least the alternating group. In the case of two substitutions the probability in favor of the symmetric group may be taken as about $\frac{3}{4}$, and in favor of the alternating, but not symmetric, group as about $\frac{1}{4}$. In order that any given substitutions may generate a group which is only a part of the $n!$ possible substitutions, very special relations are necessary, and it is highly improbable that arbitrarily chosen substitutions $s_i = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ x_{i_1} & x_{i_2} & \dots & x_{i_n} \end{pmatrix}$ should satisfy these conditions. The exception most likely to occur would be that all the given substitutions were severally equivalent to an even number of transpositions and would consequently generate the alternating group.

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Theorem (Menezes, Quick & Roney-Dougal, 2013)

If G is simple then $P(G) \geq \frac{53}{90}$ with equality if and only if $G = A_6$.

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Any more? Groups such that all proper quotients are cyclic?

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Conjecture (Breuer, Guralnick & Kantor, 2008)

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Examples: $G = S_n$ (with $T = A_n$); $G = PGL_n(q)$ (with $T = PSL_n(q)$).

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Theorem (H, 2016)

Let $g \in \mathrm{Aut}(PSp_n(q))$ then $\langle PSp_n(q), g \rangle$ is $\frac{3}{2}$-generated.

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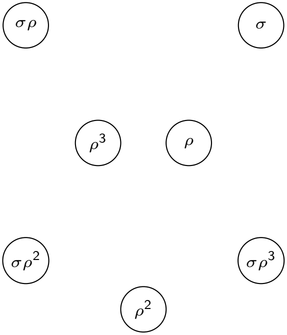

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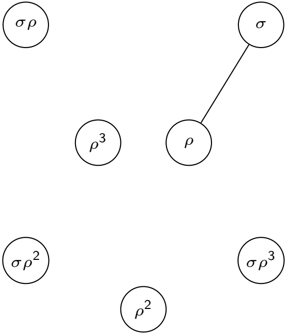

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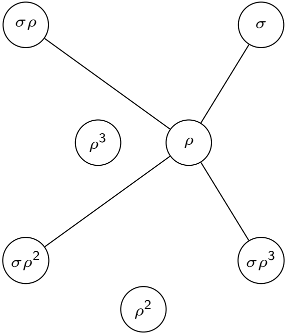

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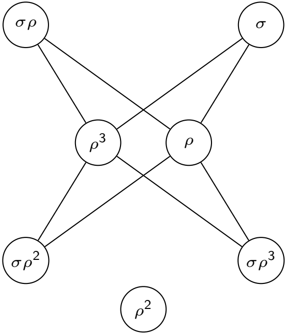

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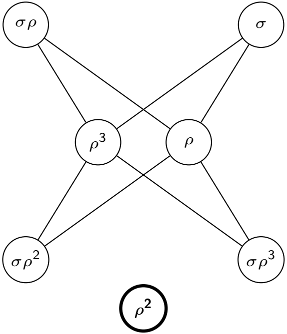

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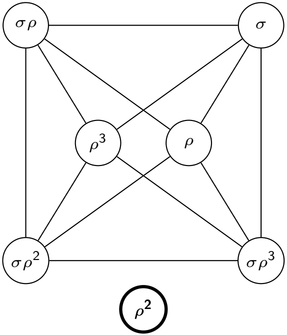

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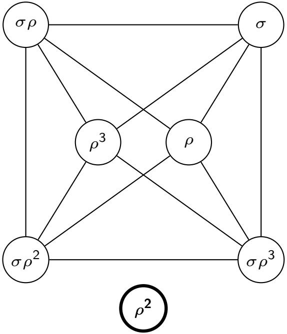

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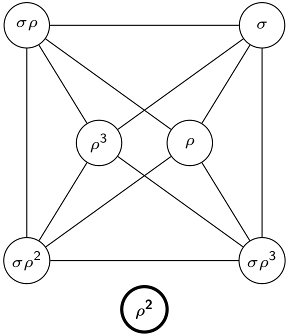

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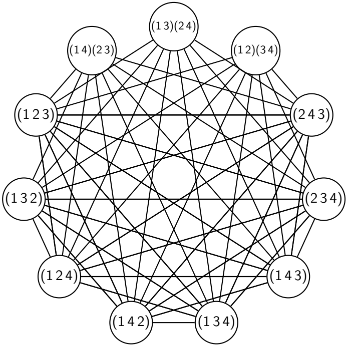

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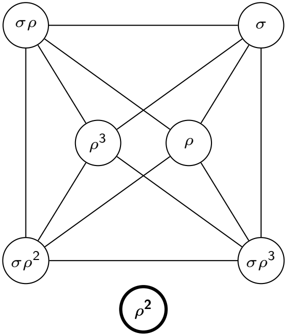

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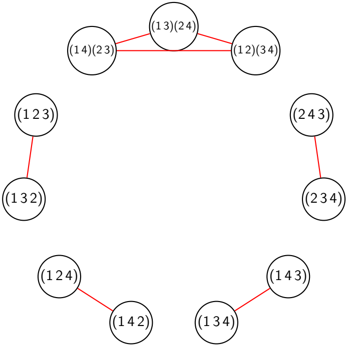

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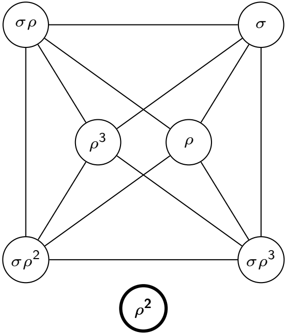

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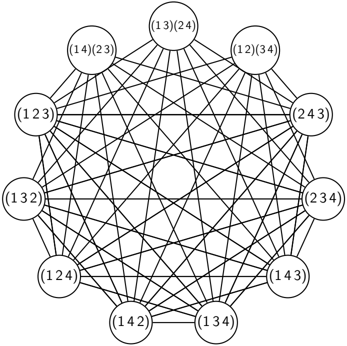

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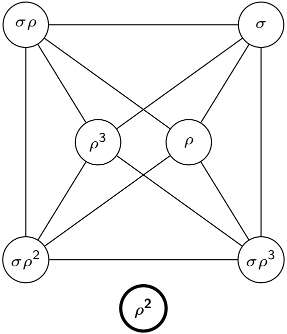

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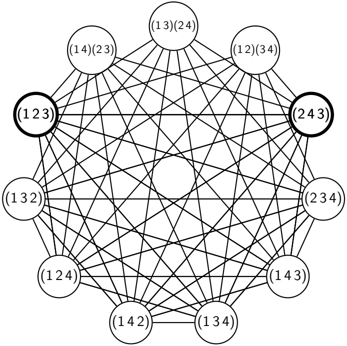

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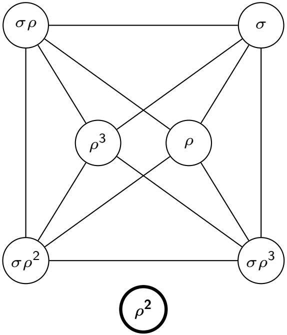

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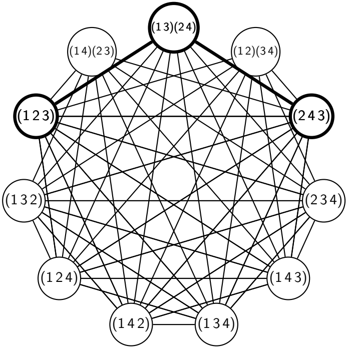

Spread

A group G has spread k if for any distinct $x_1, \dots, x_k \in G \setminus 1$ there exists an element $z \in G$ such that $\langle x_1, z \rangle = \cdots = \langle x_k, z \rangle = G$.

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Theorem (H, 2016)

Write $G = \langle T, g \rangle$ where $T = PSp_n(q)$ and $g \in \mathrm{Aut}(T)$.

Then $s(G) \geq 2$ and $s(G) \to \infty$ as $q \to \infty$.

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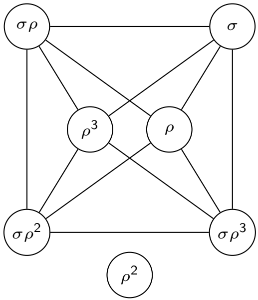

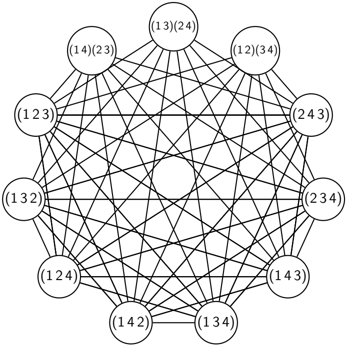

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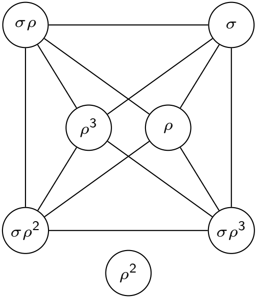

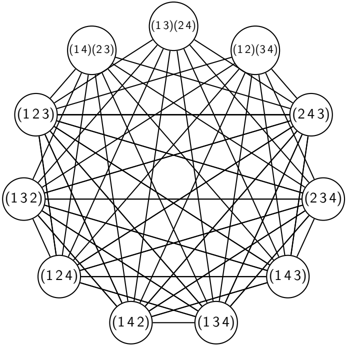

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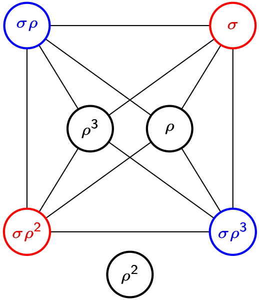

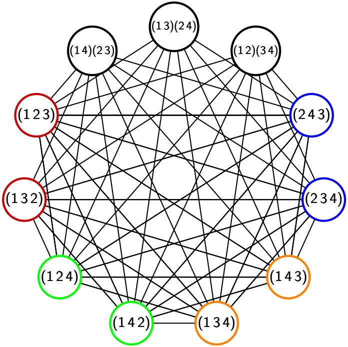

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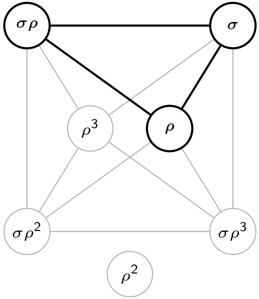

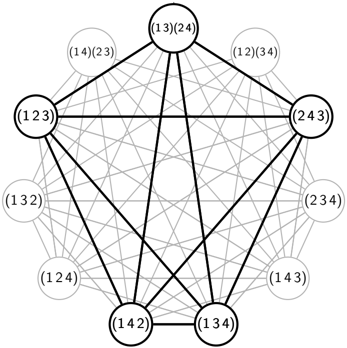

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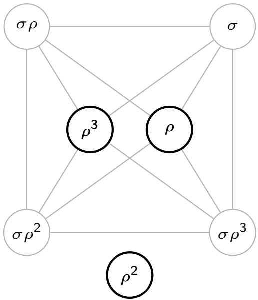

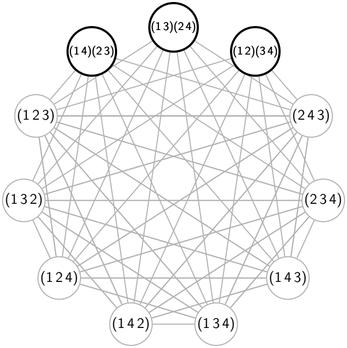

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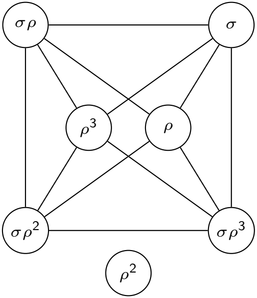

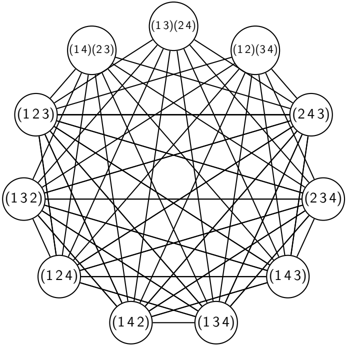

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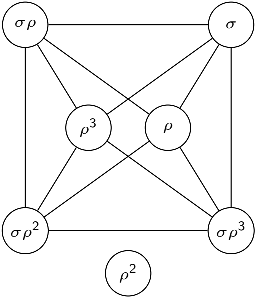

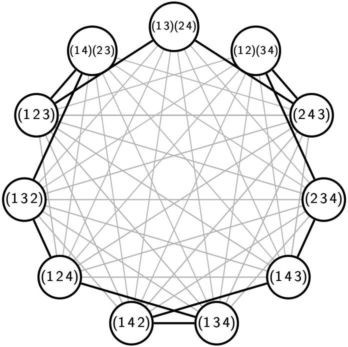

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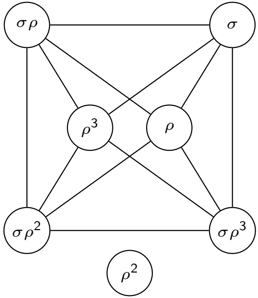

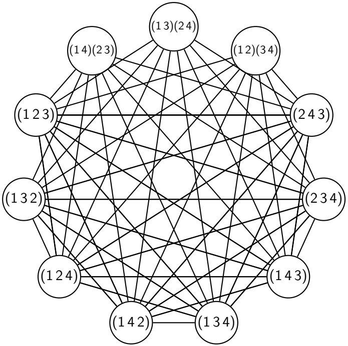

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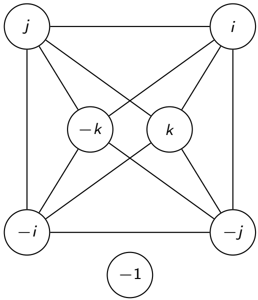

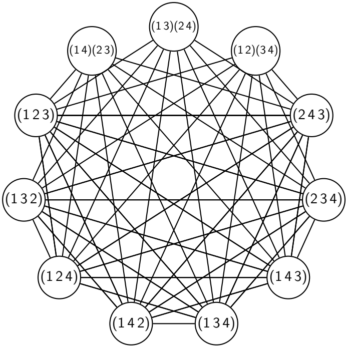

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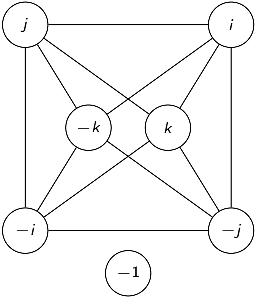
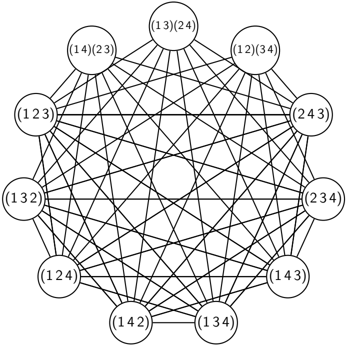
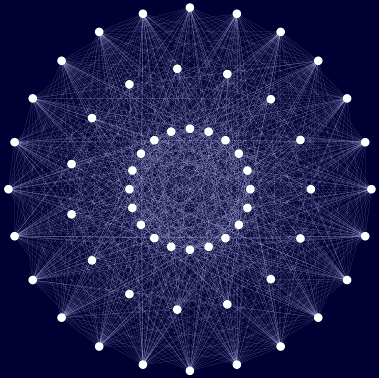




Question: Is there a finite group with spread exactly one?

$\frac{3}{2}$-Generation of Finite Groups



Scott Harper

(University of Bristol)

Young Researchers in Mathematics

2nd August 2016