Generating finite and infinite simple groups

Scott Harper

University of Bristol

Groups St Andrews

University of Newcastle

31 July 2022

the finite

Theorem (Steinberg | 1962)

Every finite simple group can be generated by two elements.

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where

$$x = (1\,2\,n)$$

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

$$x = (1\,2\,n) \qquad y = (1\,\dots\,k)(k+1\,\dots\,n)$$

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

$x = (1\,2\,n)$

$y = (1 \ldots k)(k+1 \ldots n)$

Proof

$\langle x, y \rangle$ is …

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↙ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

$x = (1\,2\,n)$ $\quad$ $y = (1 \ldots k)(k+1 \ldots n)$

Proof

$\langle x, y \rangle$ is …

transitive: x moves points between the orbits of y

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

$$x = (1\,2\,n) \qquad y = (1 \ldots k)(k+1 \ldots n)$$

Proof

$\langle x, y \rangle$ is …

transitive: x moves points between the orbits of y

primitive: k and $n - k$ are coprime [exercise]

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

$$x = (1\,2\,n) \qquad y = (1 \dots k)(k+1 \dots n)$$

Proof

$\langle x, y \rangle$ is ...

transitive: x moves points between the orbits of y

primitive: k and $n - k$ are coprime [exercise]

A_n: no other primitive subgroup has a k-cycle as $k < \frac{n}{2}$ [Marggraf | 1892]

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

~~$x = (1\,2\,n)$~~

~~$y = (1 \dots k)(k+1 \dots n)$~~

y has cycle shape $(k, n-k)$

Proof

$\langle x, y \rangle$ is …

- transitive: x moves points between the orbits of y
- primitive: k and $n-k$ are coprime [exercise]
- A_n: no other primitive subgroup has a k-cycle as $k < \frac{n}{2}$ [Marggraf | 1892]

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↙ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

~~$x = (1\,2\,n)$~~

x is nontrivial, with $a \neq ax$

~~$y = (1 \dots k)(k+1 \dots n)$~~

y has cycle shape $(k, n-k)$
& a and ax in different cycles

Proof

$\langle x, y \rangle$ is …

- transitive: x moves points between the orbits of y
- primitive: k and $n-k$ are coprime [exercise]
- A_n: no other primitive subgroup has a k-cycle as $k < \frac{n}{2}$ [Marggraf | 1892]

Theorem (Steinberg | 1962 + CFSG)

Every finite simple group can be generated by two elements.

Example

Fix even $n \geqslant 8$. Then $A_n = \langle x, y \rangle$ where ↲ $k \in \{\frac{n}{2} - 1, \frac{n}{2} - 2\}$ is odd

~~$x = (1\,2\,n)$~~

x is nontrivial, with $a \neq ax$

~~$y = (1 \ldots k)(k+1 \ldots n)$~~

y has cycle shape $(k, n-k)$

& a and ax in different cycles

Proof

$\langle x, y \rangle$ is …

transitive: x moves points between the orbits of y

primitive: k and $n-k$ are coprime [exercise]

A_n: no other primitive subgroup has a k-cycle as $k < \frac{n}{2}$ [Marggraf | 1892]

Consequence

For even $n \geqslant 8$, for all nontrivial $x \in A_n$, there exists $y \in A_n$ with $A_n = \langle x, y \rangle$.

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

Which subsets $S \subseteq G$ arise as the set of images of a word $w \in F_2$?

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

Which subsets $S \subseteq G$ arise as the set of images of a word $w \in F_2$?

If $w = x^{-1}y^{-1}xy$, then $w(G) = G$ [Liebeck, O'Brien, Shalev, Tiep | 2010].

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

Which subsets $S \subseteq G$ arise as the set of images of a word $w \in F_2$?

If $w = x^{-1}y^{-1}xy$, then $w(G) = G$ [Liebeck, O'Brien, Shalev, Tiep | 2010].

Theorem (Lubotzky | 2014)

There exists $w \in F_2$ such that $S = w(G)$ iff $1 \in S$ and $S^a = S$ for all $a \in \mathrm{Aut}(G)$.

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

Which subsets $S \subseteq G$ arise as the set of images of a word $w \in F_2$?

If $w = x^{-1}y^{-1}xy$, then $w(G) = G$ [Liebeck, O'Brien, Shalev, Tiep | 2010].

Theorem (Lubotzky | 2014)

There exists $w \in F_2$ such that $S = w(G)$ iff $1 \in S$ and $S^a = S$ for all $a \in \mathrm{Aut}(G)$.

The only CFSG-dependent ingredient of this is [Guralnick & Kantor | 2000].

A group is **$\frac{3}{2}$-generated** if every nontrivial element is in a generating pair.

Theorem (Guralnick & Kantor | 2000)

Every finite simple group is $\frac{3}{2}$-generated.

Application

Let G be a finite simple group.

Which subsets $S \subseteq G$ arise as the set of images of a word $w \in F_2$?

If $w = x^{-1}y^{-1}xy$, then $w(G) = G$ [Liebeck, O'Brien, Shalev, Tiep | 2010].

Theorem (Lubotzky | 2014)

There exists $w \in F_2$ such that $S = w(G)$ iff $1 \in S$ and $S^a = S$ for all $a \in \mathrm{Aut}(G)$.

The only CFSG-dependent ingredient of this is [Guralnick & Kantor | 2000].

Theorem (Ionescu | 1976)

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$. Then for every nonzero $x \in \mathfrak{g}$, there exists $y \in \mathfrak{g}$ such that x and y generate $\mathfrak{g}$ as a Lie algebra.

The **spread** of a group G, written $\mathbf{s(G)}$, is the greatest k such that for all nontrivial $x_1, \ldots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

The **spread** of a group G, written $s(G)$, is the greatest k such that for all nontrivial $x_1, \ldots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

The **spread** of a group G, written $s(G)$, is the greatest k such that for all nontrivial $x_1, \ldots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

The **spread** of a group G, written $\boldsymbol{s(G)}$, is the greatest k such that for all nontrivial $x_1, \ldots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

Application

The **product replacement graph**, written $\boldsymbol{\Gamma_k(G)}$, The **spread** of a group G, written $s(G)$, is the greatest k such that for all nontrivial $x_1, \dots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

Application

The **product replacement graph**, written $\Gamma_k(G)$, has as vertices the generating k-tuples of G

The **spread** of a group G, written $s(G)$, is the greatest k such that for all nontrivial $x_1, \dots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

Application

The **product replacement graph**, written $\Gamma_k(G)$, has as vertices the generating k-tuples of G and the neighbours of $(x_1, \dots, x_i, \dots, x_k)$ are $(x_1, \dots, x_i x_j^{\pm}, \dots, x_k)$ and $(x_1, \dots, x_j^{\pm} x_i, \dots, x_k)$ for $1 \leqslant i \neq j \leqslant k$.

The **spread** of a group G, written $s(G)$, is the greatest k such that for all nontrivial $x_1, \ldots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

Application

The **product replacement graph**, written $\Gamma_k(G)$, has as vertices the generating k-tuples of G and the neighbours of $(x_1, \ldots, x_i, \ldots, x_k)$ are $(x_1, \ldots, x_i x_j^{\pm}, \ldots, x_k)$ and $(x_1, \ldots, x_j^{\pm} x_i, \ldots, x_k)$ for $1 \leqslant i \neq j \leqslant k$.

Question (Pak | 2001) Is $\Gamma_k(G)$ connected for all $k > d(G)$?

The **spread** of a group G, written **$s(G)$**, is the greatest k such that for all nontrivial $x_1, \dots, x_k \in G$ there exists $y \in G$ such that $\langle x_i, y \rangle = G$.

Note that $s(G) \geqslant 1$ iff G is $\frac{3}{2}$-generated.

Theorem (Breuer, Guralnick & Kantor | 2008)

For every finite simple group G we have $s(G) \geqslant 2$.

Application

The **product replacement graph**, written **$\Gamma_k(G)$**, has as vertices the generating k-tuples of G and the neighbours of $(x_1, \dots, x_i, \dots, x_k)$ are $(x_1, \dots, x_i x_j^{\pm}, \dots, x_k)$ and $(x_1, \dots, x_j^{\pm} x_i, \dots, x_k)$ for $1 \leqslant i \neq j \leqslant k$.

Question (Pak | 2001) Is $\Gamma_k(G)$ connected for all $k > d(G)$?

Lemma (Evans | 1993)

If $s(G) \geqslant 2$, then all redundant generating k-tuples are connected in $\Gamma_k(G)$.

A generating tuple is **redundant** if a proper subtuple also generates.

Determining the spread of a group exactly is not easy!

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

- if q is even, then $s(G) = q - 2$

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

- if q is even, then $s(G) = q - 2$
- if $q \equiv 1 \pmod 4$, then $s(G) = q - 1$

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

- if q is even, then $s(G) = q - 2$
- if $q \equiv 1 \pmod 4$, then $s(G) = q - 1$
- if $q \equiv 3 \pmod 4$, then $s(G) \geqslant q - 3$.

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

- if q is even, then $s(G) = q - 2$
- if $q \equiv 1 \pmod 4$, then $s(G) = q - 1$
- if $q \equiv 3 \pmod 4$, then $s(G) \geqslant q - 3$.

Question

What is the spread of $\mathrm{PSL}_2(q)$ when $q \equiv 3 \pmod 4$?

Determining the spread of a group exactly is not easy!

Theorem (Burness & H | 2020)

Let $G = \mathrm{PSL}_2(q)$ with $q \geqslant 11$. Then

- if q is even, then $s(G) = q - 2$
- if $q \equiv 1 \pmod 4$, then $s(G) = q - 1$
- if $q \equiv 3 \pmod 4$, then $s(G) \geqslant q - 3$.

Question

What is the spread of $\mathrm{PSL}_2(q)$ when $q \equiv 3 \pmod 4$?

If $q \equiv 3 \pmod 4$ is prime, then $s(G) \geqslant \frac{1}{2}(3q - 7)$ [Burness & H | 2020].

Question (Brenner & Wiegold | 1975)

Are there any finite groups G with $s(G) = 1$?

Question (Brenner & Wiegold | 1975)

Are there any finite groups G with $s(G) = 1$?

Theorem (Brenner & Wiegold | 1975)

Let G be a finite soluble group. Then

$$s(G) \geqslant 2 \iff s(G) \geqslant 1 \iff G/N \text{ is cyclic for all } 1 \neq N \trianglelefteqslant G.$$

Question (Brenner & Wiegold | 1975)

Are there any finite groups G with $s(G) = 1$?

Theorem (Brenner & Wiegold | 1975)

Let G be a finite soluble group. Then

$$s(G) \geqslant 2 \iff s(G) \geqslant 1 \iff G/N \text{ is cyclic for all } 1 \neq N \trianglelefteq G.$$

Note For any group G, if $s(G) \geqslant 1$, then for all $1 \neq N \trianglelefteq G$ and $1 \neq n \in N$, there exists $g \in G$ such that $G = \langle n, g \rangle$, so $G/N = \langle Ng \rangle$, which is cyclic.

Question (Brenner & Wiegold | 1975)

Are there any finite groups G with $s(G) = 1$?

Theorem (Brenner & Wiegold | 1975)

Let G be a finite soluble group. Then

$$s(G) \geqslant 2 \iff s(G) \geqslant 1 \iff G/N \text{ is cyclic for all } 1 \neq N \trianglelefteqslant G.$$

Note For any group G, if $s(G) \geqslant 1$, then for all $1 \neq N \trianglelefteqslant G$ and $1 \neq n \in N$, there exists $g \in G$ such that $G = \langle n, g \rangle$, so $G/N = \langle Ng \rangle$, which is cyclic.

Theorem (Burness, Guralnick & H | 2021)

Let G be a finite group. Then

$$s(G) \geqslant 2 \iff s(G) \geqslant 1 \iff G/N \text{ is cyclic for all } 1 \neq N \trianglelefteqslant G.$$

the infinite

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

Example In $\langle \mathrm{Alt}(\mathbb{Z}), x \mapsto x+3 \rangle$ no 3-cycle is in a generating pair [Cox, 2022].

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

Example In $\langle \mathrm{Alt}(\mathbb{Z}), x \mapsto x + 3 \rangle$ no 3-cycle is in a generating pair [Cox, 2022].

Question Is there a 2-generated simple group that is not $\frac{3}{2}$-generated.

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

Example In $\langle \mathrm{Alt}(\mathbb{Z}), x \mapsto x + 3 \rangle$ no 3-cycle is in a generating pair [Cox, 2022].

Question Is there a 2-generated simple group that is not $\frac{3}{2}$-generated.

Nevertheless … can we find some infinite $\frac{3}{2}$-generated groups?

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

Example In $\langle \mathrm{Alt}(\mathbb{Z}), x \mapsto x+3 \rangle$ no 3-cycle is in a generating pair [Cox, 2022].

Question Is there a 2-generated simple group that is not $\frac{3}{2}$-generated.

Nevertheless … can we find some infinite $\frac{3}{2}$-generated groups?

Observation (H | 2020)
Let G be a infinite soluble group such that every proper quotient is cyclic. Then G is $\frac{3}{2}$-generated

What can we say about $\frac{3}{2}$-generation of infinite simple groups?

Some simple groups are not finitely generated.

Example The alternating group $\mathrm{Alt}(\mathbb{Z})$ is locally finite.

Some finitely generated simple groups are not 2-generated.

Example The Guba group: a finitely generated simple group where every 2-generated subgroup is free [Guba, 1986].

Some 2-generated groups with only cyclic quotients are not $\frac{3}{2}$-generated.

Example In $\langle \mathrm{Alt}(\mathbb{Z}), x \mapsto x+3 \rangle$ no 3-cycle is in a generating pair [Cox, 2022].

Question Is there a 2-generated simple group that is not $\frac{3}{2}$-generated.

Nevertheless … can we find some infinite $\frac{3}{2}$-generated groups?

Observation (H | 2020)
Let G be a infinite soluble group such that every proper quotient is cyclic.
Then G is $\frac{3}{2}$-generated … since G is cyclic!

Thompson's group *V*: first known finitely presented infinite simple group.

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0, 1\}^{\mathbb{N}}$.

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$.

$$[0,1] =$$

Thompson's group *V*: first known finitely presented infinite simple group.

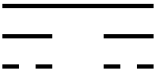
Let $\mathfrak{C}$ be the Cantor space $\{0, 1\}^{\mathbb{N}}$.

$[0, 1] =$ 

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0, 1\}^{\mathbb{N}}$.

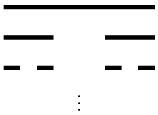
$[0, 1] =$



Thompson's group *V*: first known finitely presented infinite simple group.

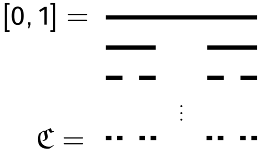
Let $\mathfrak{C}$ be the Cantor space $\{0, 1\}^{\mathbb{N}}$.

$[0, 1] =$



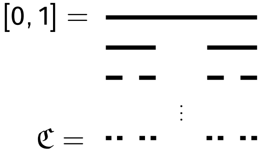
Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$.



Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.



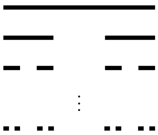
Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.

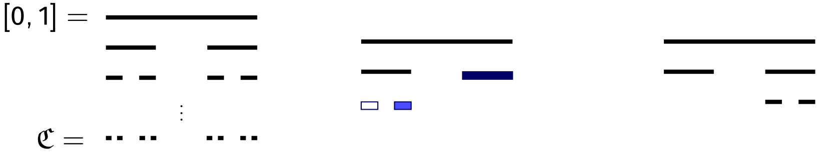
$[0,1] =$



$\mathfrak{C} =$

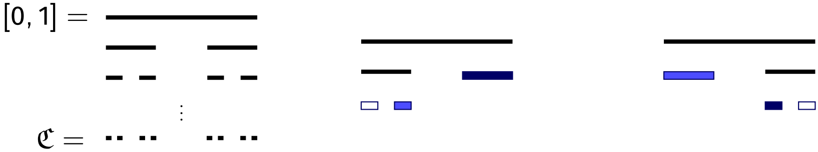
Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.



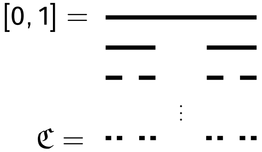
Thompson's group *V*: first known finitely presented infinite simple group.

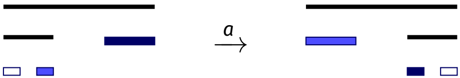
Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.



Thompson's group *V*: first known finitely presented infinite simple group.

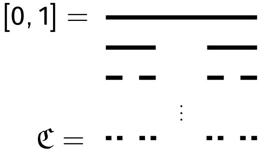
Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.

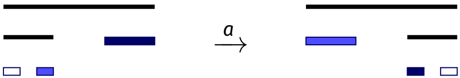




Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.





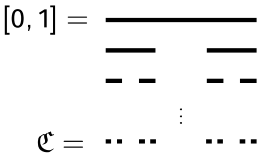
selfsimilarity

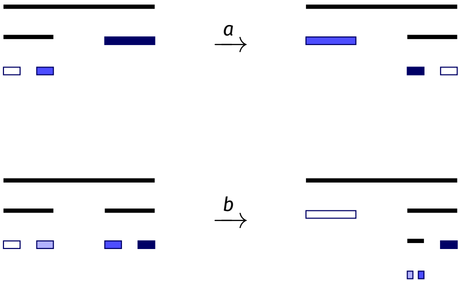
+

symmetric group

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.





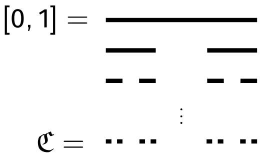
selfsimilarity

+

symmetric group

Thompson's group *V*: first known finitely presented infinite simple group.

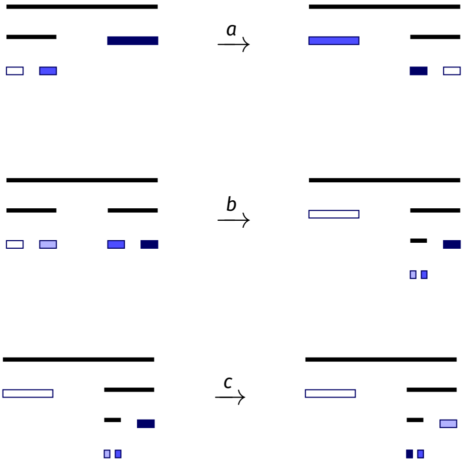
Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.



selfsimilarity

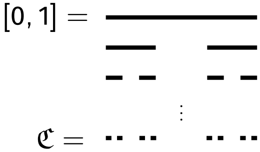
+

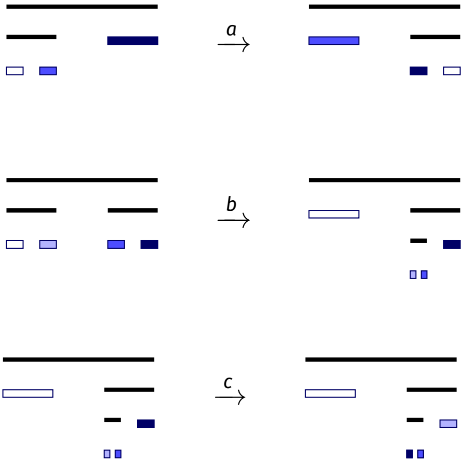
symmetric group



Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.





selfsimilarity

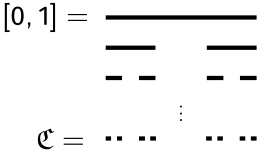
+

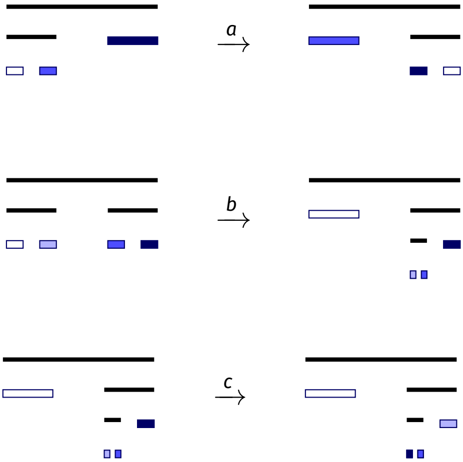
symmetric group

$c = (100\ 11)$

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.





selfsimilarity

+

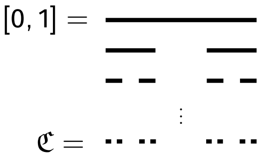
symmetric group

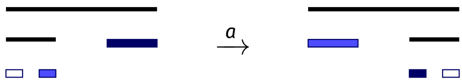
a **transposition**

$c = (100\ 11)$

Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.

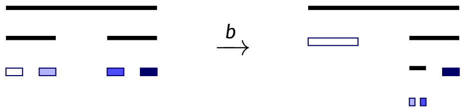




selfsimilarity

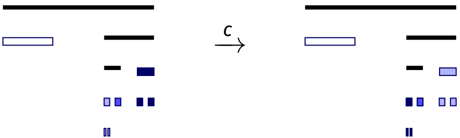
+

symmetric group



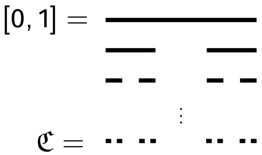
a **transposition**

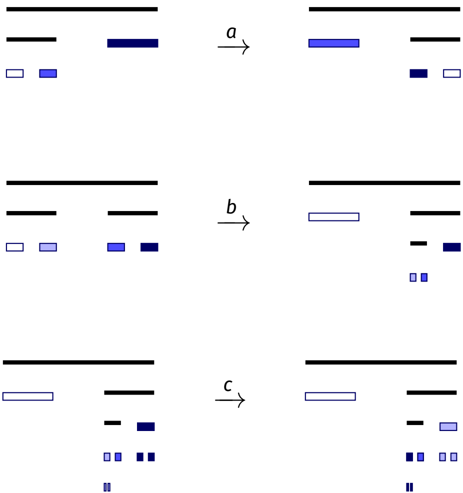
$c = (100\ 11)$



Thompson's group *V*: first known finitely presented infinite simple group.

Let $\mathfrak{C}$ be the Cantor space $\{0,1\}^{\mathbb{N}}$. Then V acts on $\mathfrak{C}$ by homeomorphisms.





selfsimilarity

+

symmetric group

a **transposition**

$c = (100\ 11)$

$c = (1000\ 110)(1001\ 111)$

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\, j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W \text{ disjoint basic open sets of } \mathfrak{C} \rangle$ [Brin '04]

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\, j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W \text{ disjoint basic open sets of } \mathfrak{C} \rangle$ [Brin '04]

$V = \langle t_{U,W} \mid t_{U,W}^2,\ t_{U,W}^{t_{X,Y}} = t_{U(X\,Y),W(X\,Y)},\ t_{U,W} = t_{U0,W0}t_{U1,W1} \rangle$ [Bleak & Quick '17]

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j$ distinct elements of $\Omega\rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)}\rangle$

For $A \subseteq \Omega$, write $G_{[A]}$ for the pointwise stabiliser of $\Omega \setminus A$. Then $G_{[A]} \cong S_{|A|}$.

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W$ disjoint basic open sets of $\mathfrak{C}\rangle$ [Brin ’04]

$V = \langle t_{U,W} \mid t_{U,W}^2,\ t_{U,W}^{t_{X,Y}} = t_{U(X\,Y),W(X\,Y)},\ t_{U,W} = t_{U0,W0}t_{U1,W1}\rangle$ [Bleak & Quick ’17]

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

For $A \subseteq \Omega$, write $G_{[A]}$ for the pointwise stabiliser of $\Omega \setminus A$. Then $G_{[A]} \cong S_{|A|}$.

Covering Lemma for S_n

Let $A_1, \ldots, A_k \subseteq \Omega$ such that $A_i \cap A_{i+1} \neq \emptyset$. Then $G = \langle G_{[A_1]}, \ldots, G_{[A_k]} \rangle$.

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W \text{ disjoint basic open sets of } \mathfrak{C} \rangle$ [Brin '04]

$V = \langle t_{U,W} \mid t_{U,W}^2,\ t_{U,W}^{t_{X,Y}} = t_{U(X\,Y),W(X\,Y)},\ t_{U,W} = t_{U0,W0} t_{U1,W1} \rangle$ [Bleak & Quick '17]

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

For $A \subseteq \Omega$, write $G_{[A]}$ for the pointwise stabiliser of $\Omega \setminus A$. Then $G_{[A]} \cong S_{|A|}$.

Covering Lemma for S_n

Let $A_1, \ldots, A_k \subseteq \Omega$ such that $A_i \cap A_{i+1} \neq \emptyset$. Then $G = \langle G_{[A_1]}, \ldots, G_{[A_k]} \rangle$.

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W \text{ disjoint basic open sets of } \mathfrak{C} \rangle$ [Brin '04]

$V = \langle t_{U,W} \mid t_{U,W}^2,\ t_{U,W}^{t_{X,Y}} = t_{U(X\,Y),W(X\,Y)},\ t_{U,W} = t_{U0,W0} t_{U1,W1} \rangle$ [Bleak & Quick '17]

For clopen $U \subseteq \mathfrak{C}$, write $V_{[U]}$ for the p/wise stabiliser of $\mathfrak{C} \setminus U$. Then $V_{[U]} \cong V$.

Let $G = S_n$ and $\Omega = \{1, \ldots, n\}$.

$G = \langle (i\,j) : i, j \text{ distinct elements of } \Omega \rangle$

$G = \langle t_{i,j} \mid t_{i,j}^2,\ t_{i,j}^{t_{k,l}} = t_{i(k\,l),j(k\,l)} \rangle$

For $A \subseteq \Omega$, write $G_{[A]}$ for the pointwise stabiliser of $\Omega \setminus A$. Then $G_{[A]} \cong S_{|A|}$.

Covering Lemma for S_n

Let $A_1, \ldots, A_k \subseteq \Omega$ such that $A_i \cap A_{i+1} \neq \emptyset$. Then $G = \langle G_{[A_1]}, \ldots, G_{[A_k]} \rangle$.

Let $(U\,W) \in V$ transpose the two disjoint basic open sets $U, W \subseteq \mathfrak{C}$.

$V = \langle (U\,W) : U, W \text{ disjoint basic open sets of } \mathfrak{C} \rangle$ [Brin '04]

$V = \langle t_{U,W} \mid t_{U,W}^2,\ t_{U,W}^{t_{X,Y}} = t_{U(X\,Y),W(X\,Y)},\ t_{U,W} = t_{U0,W0}t_{U1,W1} \rangle$ [Bleak & Quick '17]

For clopen $U \subseteq \mathfrak{C}$, write $V_{[U]}$ for the p/wise stabiliser of $\mathfrak{C} \setminus U$. Then $V_{[U]} \cong V$.

Covering Lemma for V

Let $U_1, \ldots, U_k \subseteq \mathfrak{C}$ be clopen s.t. $U_i \cap U_{i+1} \neq \emptyset$. Then $V = \langle V_{[U_1]}, \ldots, V_{[U_k]} \rangle$.

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

Brin–Thompson groups nV which act on the product space $\mathfrak{C}^n$

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

- **Brin–Thompson groups nV** which act on the product space $\mathfrak{C}^n$
- **Higman–Thompson groups V_n** and V_n' which act on $\mathfrak{C}_n = \{1, \ldots, n\}^{\mathbb{N}}$

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

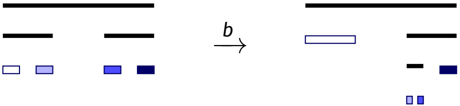
- **Brin–Thompson groups nV** which act on the product space $\mathfrak{C}^n$
- **Higman–Thompson groups V_n** and V_n' which act on $\mathfrak{C}_n = \{1, \ldots, n\}^{\mathbb{N}}$

Theorem (Donoven & H | 2020)

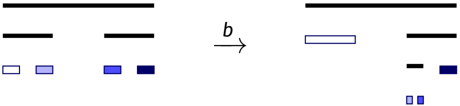
For all $n \geqslant 2$, the groups V_n, V_n' and nV are $\frac{3}{2}$-generated.

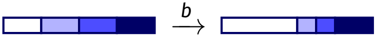
$\mathbf{F} = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

$\color{red}{F} = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$



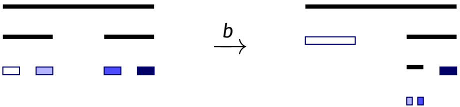
$F = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

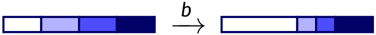




as a homeomorphism of $[0, 1]$

$F = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

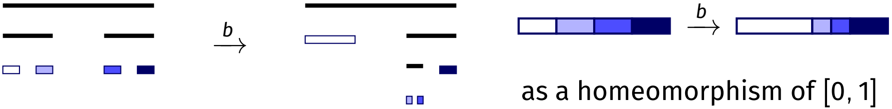




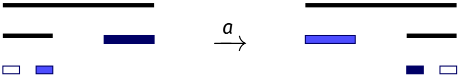
as a homeomorphism of $[0, 1]$

$T = \{g \in V \mid \text{permutation associated to } g \text{ is cyclic}\}$

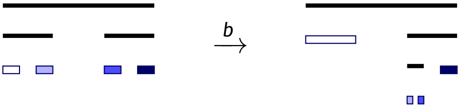
$\textbf{F} = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

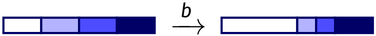


$\textbf{T} = \{g \in V \mid \text{permutation associated to } g \text{ is cyclic}\}$



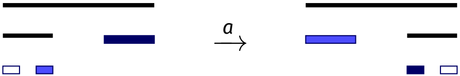
$F = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

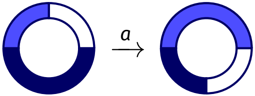




as a homeomorphism of $[0, 1]$

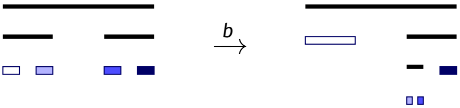
$T = \{g \in V \mid \text{permutation associated to } g \text{ is cyclic}\}$

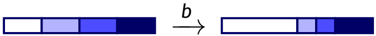




as a homeomorphism of $\mathbb{S}^1$

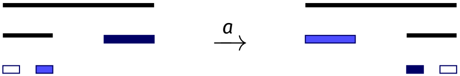
$F = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

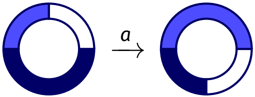




as a homeomorphism of $[0, 1]$

$T = \{g \in V \mid \text{permutation associated to } g \text{ is cyclic}\}$



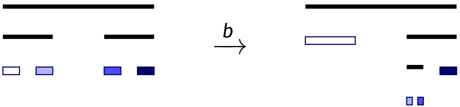


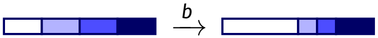
as a homeomorphism of $\mathbb{S}^1$

Theorem (Thompson | 1965 // Mason | 1977)

Thompson's groups V and T are simple, 2-generated and finitely presented.

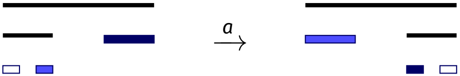
$\textcolor{red}{F} = \{g \in V \mid \text{permutation associated to } g \text{ is trivial}\}$

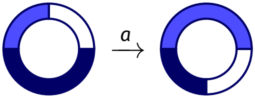




as a homeomorphism of $[0,1]$

$\textcolor{red}{T} = \{g \in V \mid \text{permutation associated to } g \text{ is cyclic}\}$





as a homeomorphism of $\mathbb{S}^1$

Theorem (Thompson | 1965 // Mason | 1977)

Thompson's groups V and T are simple, 2-generated and finitely presented. While F' is simple, $F/F' \cong \mathbb{Z}^2$; F is 2-generated but F' is not finitely generated.

Let $[a, b] \subseteq [0, 1]$ be a dyadic interval (i.e. the endpoints a, b are in $\mathbb{Z}[\frac{1}{2}]$). Write $F_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $F_{[a,b]} \cong F$.

Let $[a, b] \subseteq [0, 1]$ be a dyadic interval (i.e. the endpoints a, b are in $\mathbb{Z}[\frac{1}{2}]$). Write $F_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $F_{[a,b]} \cong F$.

Covering Lemma for F

Let $[a_1, b_1], \ldots, [a_k, b_k] \subseteq [0, 1]$ be dyadic intervals with $\bigcup_{i=1}^{k}(a_i, b_i) = (0, 1)$. Then $F = \langle F_{[a_1,b_1]}, \ldots, F_{[a_k,b_k]} \rangle$.

Let $[a, b] \subseteq [0, 1]$ be a dyadic interval (i.e. the endpoints a, b are in $\mathbb{Z}[\frac{1}{2}]$).
Write $F_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $F_{[a,b]} \cong F$.

Covering Lemma for F

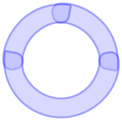
Let $[a_1, b_1], \ldots, [a_k, b_k] \subseteq [0, 1]$ be dyadic intervals with $\bigcup_{i=1}^{k}(a_i, b_i) = (0, 1)$.
Then $F = \langle F_{[a_1,b_1]}, \ldots, F_{[a_k,b_k]} \rangle$.

Let $[a, b] \subseteq \mathbb{S}$ be a dyadic interval (by making the identification $\mathbb{S} = \mathbb{R}/\mathbb{Z}$).
Write $T_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $T_{[a,b]} \cong F$.

Let $[a, b] \subseteq [0, 1]$ be a dyadic interval (i.e. the endpoints a, b are in $\mathbb{Z}[\frac{1}{2}]$). Write $F_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $F_{[a,b]} \cong F$.

Covering Lemma for F

Let $[a_1, b_1], \ldots, [a_k, b_k] \subseteq [0, 1]$ be dyadic intervals with $\bigcup_{i=1}^{k}(a_i, b_i) = (0, 1)$. Then $F = \langle F_{[a_1,b_1]}, \ldots, F_{[a_k,b_k]} \rangle$.



Let $[a, b] \subseteq \mathbb{S}$ be a dyadic interval (by making the identification $\mathbb{S} = \mathbb{R}/\mathbb{Z}$). Write $T_{[a,b]}$ for the pointwise stabiliser of $[0, 1] \setminus (a, b)$. Then $T_{[a,b]} \cong F$.

Covering Lemma for T

Let $[a_1, b_1], \ldots, [a_k, b_k] \subseteq \mathbb{S}^1$ be dyadic intervals with $\bigcup_{i=1}^{k}(a_i, b_i) = \mathbb{S}^1$. Then $T = \langle T_{[a_1,b_1]}, \ldots, T_{[a_k,b_k]} \rangle$.

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

- **Brin–Thompson groups nV** which act on the product space $\mathfrak{C}^n$
- **Higman–Thompson groups V_n** and V_n' which act on $\mathfrak{C}_n = \{1, \ldots, n\}^{\mathbb{N}}$

Theorem (Donoven & H | 2020)

For all $n \geqslant 2$, the groups V_n, V_n' and nV are $\frac{3}{2}$-generated.

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

- **Brin–Thompson groups nV** which act on the product space $\mathfrak{C}^n$
- **Higman–Thompson groups V_n** and V_n' which act on $\mathfrak{C}_n = \{1, \ldots, n\}^{\mathbb{N}}$

Theorem (Donoven & H | 2020)

For all $n \geqslant 2$, the groups V_n, V_n' and nV are $\frac{3}{2}$-generated.

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

Theorem (Donoven & H | 2020)

Thompson's group V is $\frac{3}{2}$-generated.

Our methods apply to various generalisations of V:

- **Brin–Thompson groups nV** which act on the product space $\mathfrak{C}^n$
- **Higman–Thompson groups V_n** and V_n' which act on $\mathfrak{C}_n = \{1, \ldots, n\}^{\mathbb{N}}$

Theorem (Donoven & H | 2020)

For all $n \geqslant 2$, the groups V_n, V_n' and nV are $\frac{3}{2}$-generated.

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

Lots more work in progress with Bleak, Donoven, Hyde & Skipper.

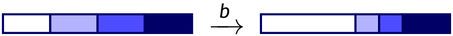
Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

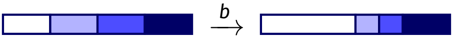
We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)

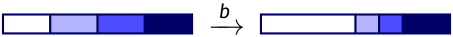


Proof idea

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



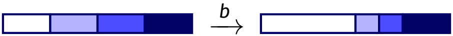
Proof idea

Let $x \in T$ be nontrivial.

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



Proof idea

Let $x \in T$ be nontrivial.

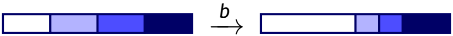
$|x|$ finite:

$|x|$ infinite:

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

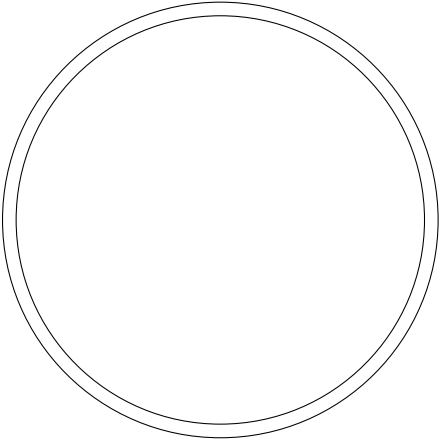
We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)

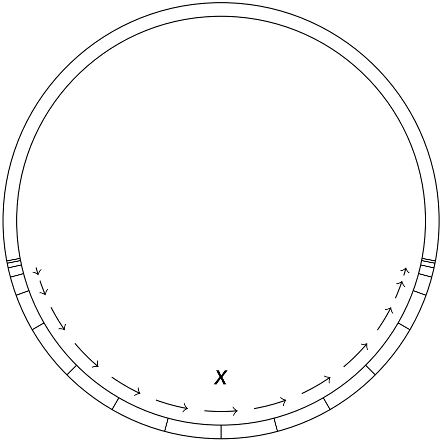


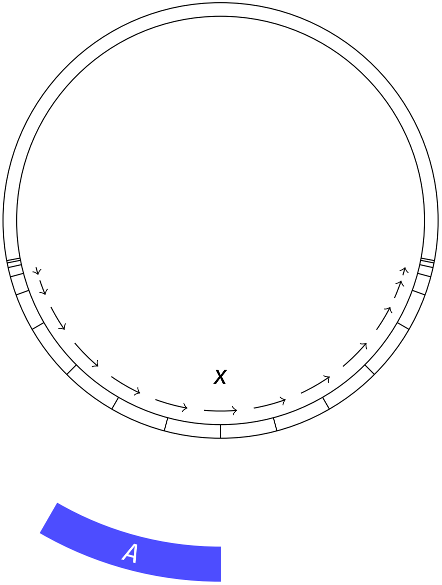
Proof idea

Let $x \in T$ be nontrivial.

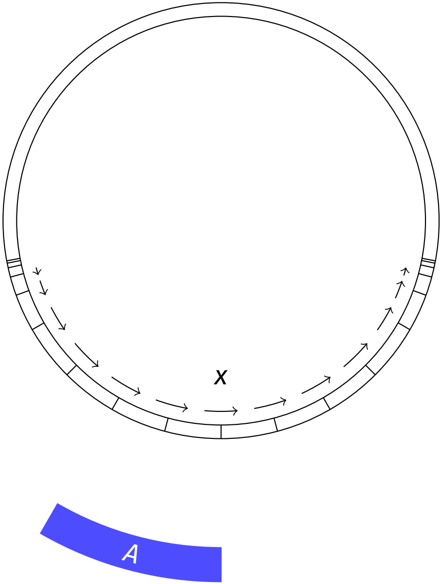
- $|x|$ finite: we can give x explicitly and use a combinatorial generation criterion from [Golan | 2021] based on the Stallings 2-core
- $|x|$ infinite:



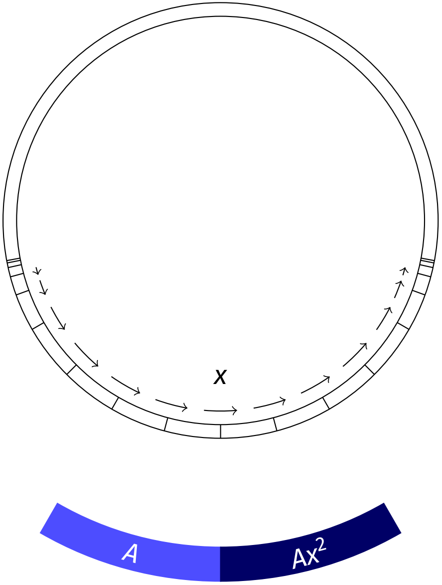

X



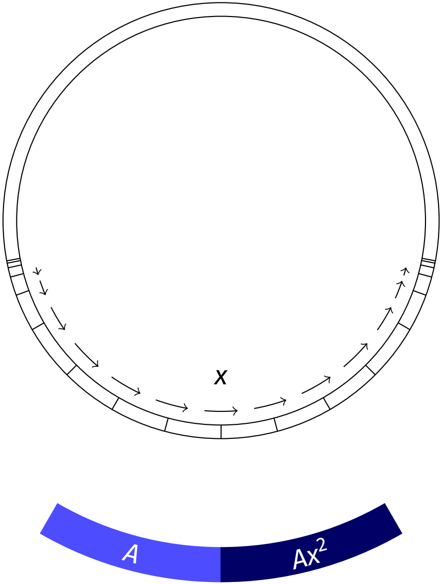
X
A



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

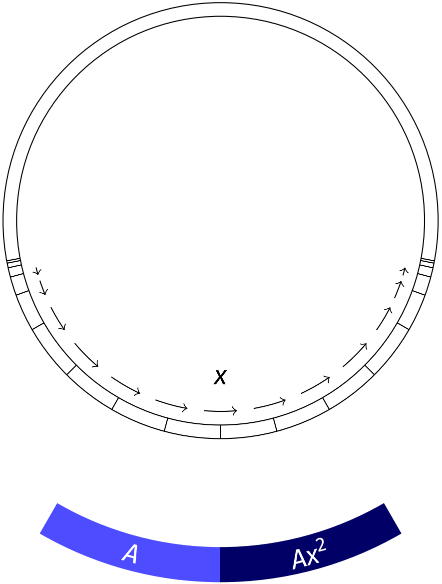


Fix $T_{[A]} = \langle a_0, a_1 \rangle$.



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

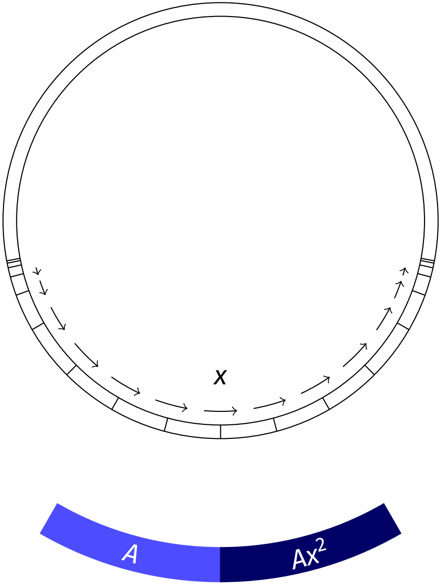
Let $y = x a_0 a_1^{x^2}$.



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

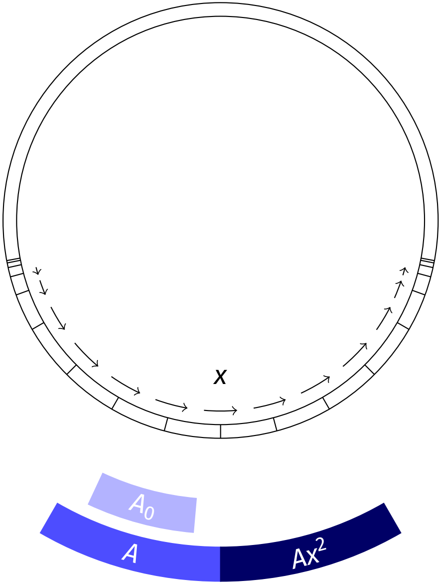
So $\langle x, y \rangle \geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle'$



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

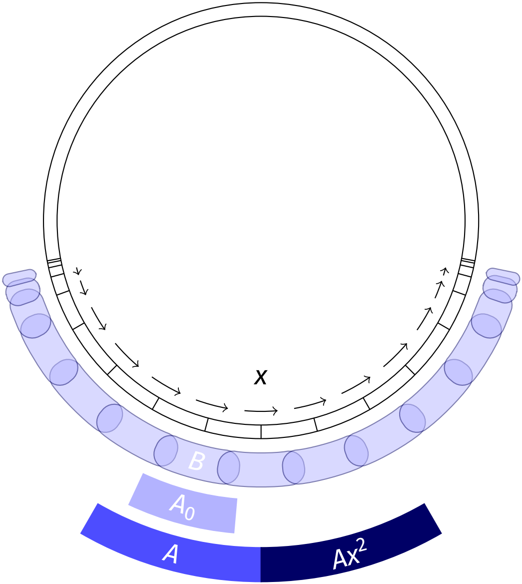
So $\langle x, y \rangle \geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle'$
$\geqslant (T_{[A]})'$



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

$$\begin{aligned} \text{So } \langle x, y \rangle &\geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle' \\ &\geqslant (T_{[A]})' \\ &\geqslant T_{[A_0]}. \end{aligned}$$

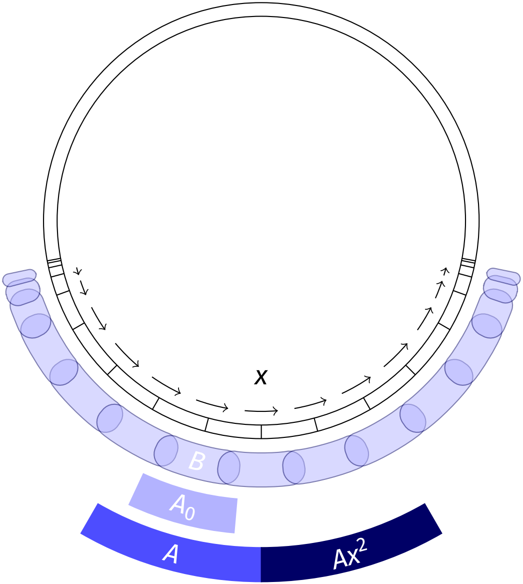


Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

$$\begin{aligned} \text{So } \langle x, y\rangle &\geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle' \\ &\geqslant (T_{[A]})' \\ &\geqslant T_{[A_0]}. \end{aligned}$$

So $\langle x, y\rangle \geqslant \langle T_{[A_0]}, x\rangle \geqslant T_{[B]}$ where $B = \cup_{i\in\mathbb{Z}} A_0 x^i$.



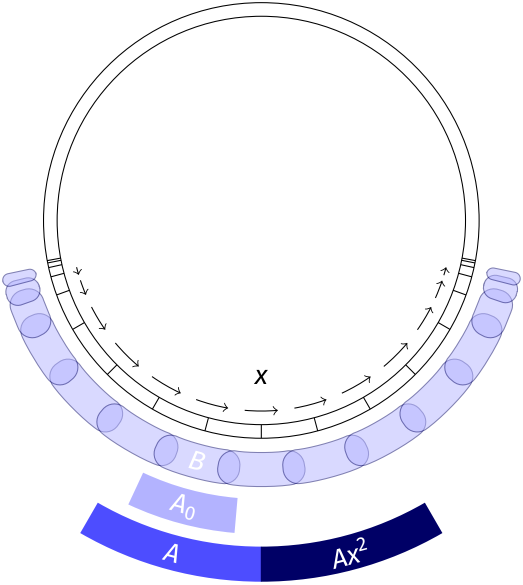
Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

So $\langle x, y \rangle \geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle'$
$\geqslant (T_{[A]})'$
$\geqslant T_{[A_0]}$.

So $\langle x, y \rangle \geqslant \langle T_{[A_0]}, x \rangle \geqslant T_{[B]}$
where $B = \cup_{i \in \mathbb{Z}} A_0 x^i$.

Tweak y to handle $\mathbb{S}^1 \setminus B$.



Fix $T_{[A]} = \langle a_0, a_1 \rangle$.

Let $y = x a_0 a_1^{x^2}$.

So $\langle x, y \rangle \geqslant \langle a_0 a_1^{x^2}, a_0^{x^{-2}} a_1 \rangle'$
$\geqslant (T_{[A]})'$
$\geqslant T_{[A_0]}$.

So $\langle x, y \rangle \geqslant \langle T_{[A_0]}, x \rangle \geqslant T_{[B]}$
where $B = \cup_{i \in \mathbb{Z}} A_0 x^i$.

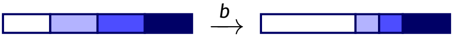
Tweak y to handle $\mathbb{S}^1 \setminus B$.

Show that every infinite order elt (including b) is conjugate to an elt like y.

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



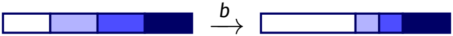
Let $x \in T$ be nontrivial.

- $|x|$ finite: we can give x explicitly and use a combinatorial generation criterion from [Golan | 2021] based on the Stallings 2-core
- $|x|$ infinite:

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



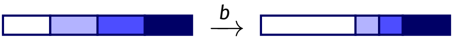
Let $x \in T$ be nontrivial.

- $|x|$ finite: we can give x explicitly and use a combinatorial generation criterion from [Golan | 2021] based on the Stallings 2-core
- $|x|$ infinite: we build y based on standard generators of F and orbits of x and use a dynamical argument to make this a conjugate of b

Theorem (Bleak, H & Skipper | 2022)

Thompson's group T is $\frac{3}{2}$-generated.

We show that for all nontrivial $x \in T$ there exists $y \in b^T$ such that $\langle x, y \rangle$.
(Recall the A_n example 45 mins ago.)



Let $x \in T$ be nontrivial.

- $|x|$ finite: we can give x explicitly and use a combinatorial generation criterion from [Golan | 2021] based on the Stallings 2-core
- $|x|$ infinite: we build y based on standard generators of F and orbits of x and use a dynamical argument to make this a conjugate of b

Theorem (Bleak, H & Skipper | 2022)

For any $s, t \in T$ of infinite order, there exists $g \in T$ such that $\langle s, t^g \rangle = T$.